Unit - 1

Relations & functions

cartesian product

Let A & B Be 2 sets then, the cartesian product of A & B is denoted by $A \times B$ is set of all ordered pairs (a, b)

$A \times B : \{(a,b) \mid a \in A, b \in B\}$

Example 1

If $A : \{a, b, c\}$ $B : \{1, 2\}$ compute

Solution

$A \times B : \{(a,1)(a,2)(b,1)(b,2)(c,1)(c,2)\}$

$B \times A : \{(1,a)(1,b)(1,c)(2,a)(2,b)(2,c)\}$

$A \times A : \{(a,a)(a,b)(a,c)(b,a)(b,b)(b,c)(c,a)(c,b)(c,c)\}$

$B \times B : \{(1,1)(1,2)(2,1)(2,2)\}$

Example 2

$A : \{2, 3, 4\}$ $B : \{a, b, c\}$

$B \times A : \{(a,2)(a,3)(a,4)(b,2)(b,3)(b,4)(c,2)(c,3)(c,4)\}$

Binary relation

Let $A \times B$ be any 2 sets. A binary relation R from A to B is a subset of $A \times B$. $(a,b) \in R$ is also written as aRB

Example

Let $A = \{a, b, c\}$ Then $R = \{(a,a)(a,b)(c,a)\}$ is a relation in A

Inverse relation

Let R be a relation from A to B Then as inverse relation R^{-1} is defined as $R^{-1} = \{(b,a) / (a,b) \in R\}$

Types of relation

i) Reflexive
ii) Symmetric
iii) Transitive

i) Reflexive (a,a)

$: (1,1)\ (2,2)$

$(a,b)\ (b,a)$

ii) Symmetric $(1,2)\ (2,1)$

$(a,b)\ (b,c) \Rightarrow (a,c)$

iii) Transitive $(1,2)\ (2,3) \Rightarrow (1,3)$

i) Reflexive

A binary relation R in a set A is said to be reflexive if aRa (ie) $(a,a) \in R$

example

Let $A = \{1, 2, 3\}$

Then $R_1 = \{(1,1)\ (2,2)\ (3,3)\}$.

ii) Symmetric

A binary relation R in a set A is called symmetric relation if $aRb \Rightarrow bRa$

for all $a, b \in A$

ie $(a,b) \in R \Rightarrow (b,a) \in R$

Example

Let $A = \{2, 3, 4\}$

$R_2 = \{(2,3)(3,2)(3,4)(4,3)\}$

iii) **Transitive**

A binary relation R in a set A is said to be Transitive if $aRb, bRc \Rightarrow aRc$ for all $a, b, c \in A$

Example

Let $A = \{1, 2, 3\}$

Then, $R_1 = \{(1,2)(2,3)(1,3)\}$

Anti Symmetric

A binary relation R in a set A is called Anti symmetric relation if $aRb, bRa \Rightarrow a = b$ $a, b \in A$

(ie) $(a,b) \in R, (b,a) \in R \Rightarrow a = b$

1/3/21 **Equivalence Relation:-**

A Relation R defined in a set A is said to be equivalence relation if R is reflexive, symmetric and Transitive

1. If $A = \{3, 5, 7, 9, 11\}$ $B = \{2, 6, 8, 10\}$

express the relation R.

i) R is one less than : $\{(5,6)(7,8)(9,10)\}$

ii) R is a multiple of : $\{(3,6)(5,10)\}$

iii) a & b are relatively prime $\{(3,2)(5,2)(7,2)(11,2)\}$

2) If $X = \{1,2,3,4,5,6,7\}$

if $R = \{(x,y) / x-y$ is divisible by $3\}$

Show that R is equivalence relation

i) Reflexive relation

$R_1 = \{(1,1)(2,2)(3,3)(4,4)(5,5),(6,6)(7,7)\}$

$R = \{(1,4)(4,1)(2,5)(5,2)(3,6)(6,3)(4,7)(7,4)$
$(7,1)(1,7)\}$

ii) ~~Transitive~~ R satisfied Symmetric relation

$R_2 = \{(1,4)(4,1)(2,5)(5,2)(3,6)(6,3)(4,7)(7,4)$
$(7,1)(1,7)\}$

iii) R satisfied Transitive relation

$R_3 = \{(1,4)(4,1) : (1,1)$
$(2,5)(5,2) : (2,2)$
$(3,6)(6,3) : (3,3)$
$(4,7)(7,4) : (4,4)$
$(1,4)(4,7) : (1,7)$
$(7,1)(1,7) : (7,7)\}$

Example 2

if R and R' are relation in a set A.

prove

i) if R and R' are reflexive then $R \cup R'$ and $R \cap R'$ are both reflexive

ii) if R is symmetric, R' is symmetric then

$R \cup R'$ and $R \cap R'$ are both symmetric.

iii) If R and R' are transitive, then $R \cap R'$ is Transitive

Solution

i) Since R, R' are reflexive $(a,a) \in R$ and $(a,a) \in R'$ for all $a \in A$ (i,e) $(a,a) \in R \cup R'$ and $(a,a) \in R \cap R'$ for all $a \in A$

ii) $(a,b) \in R \cup R'$: $(a,b) \in R$ or $(a,b) \in R'$

: $(b,a) \in R$ or $(b,a) \in R'$

(R & R' are Symmetric)

: $(b,a) \in R \cup R'$ for all $a, b \in A$

$R \cup R'$ is symmetric, similarly it can be provided that $R \cap R'$ is also symmetric

iii) $(a,b) \in R \cap R'$ $(b,c) \in R \cap R'$

$\Rightarrow$ $[(a,b) \in R$ (or) $(a,b) \in R']$

and $[(b,c) \in R$ (or) $(b,c) \in R']$

$\Rightarrow$ $[(a,b) \in R$ (or) $(b,c) \in R]$

and $[(b,c) \in R$ or $(b,c) \in R']$

$\Rightarrow$ $(a,c) \in R$ or $(a,c) \in R'$

[$\because R$ & R' are Transitive]

$\Rightarrow$ $(a,c) \in R \cap R'$ for all a, b, c in A

$\therefore R \cap R'$ is Transitive.

Note

if R and R' are equivalence relation in A, so is

ii) if R and R' are equivalence relation in a set A, then R∪R' is not an equivalence relation

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Example 3

Show that the relation is defined in N x N by (a,b) is (c,d) if only if (a+d = b+c) is an equivalence relation

Solution

$N \times N = \{(a,b)\ (a,b) \in N\}$.

i) $(a,b) \sim (a,b)$ for all $(a,b) \in N \times N$ since $a+d = d+a$
$\therefore$ ~~$(c,d) \sim (a,b)$~~ Hence is ~~symmetric~~ reflexive

ii) if $(a,b) \sim (c,d)$ then $a+d = b+c$ ie $c+b = d+a$
$\therefore (c,d) \sim (a,b)$ Hence is ~~refle~~ symmetric

iii) if $(a,b) \sim (c,d)$ and $(c,d) \sim (e,f)$ then $a+d = b+c$ and $c+f = d+e$
$\therefore a+d+c+f = b+c+d+e$ or $a+f = b+e$

Hence $\sim$ is an equivalence relation

Example 4

Let R be the relation in the natural number N defined by $R = \{(x,y) \mid x \in N, y \in N, (x-y)$ is divisible by 5 $\}$

prove that R is an equivalence relation

Solution :-

Let $a \in N$ then $a-a = 0$ is divisible by 5 and hence $(a,a) \in R$. Thus R is reflexive
Let $(a,b) \in R$ Then $(a-b)$ is divisible by 5

and hence $(b-a) = -(a-b)$ is also divisible by 5
Then (b,a) belongs to R. R is symmetric.

Let $(a,b) \in R$ and $(b,c) \in R$ then $(a-b)$ & $(b-c)$ are each divisible by 5

(ie) (a,c) belongs to R. R is Transitive

Since R is reflexive symmetric & Transitive
R is by definition, an equivalence relation

Example 5

Consider the set $N \times N$ (ie) the set of ordered pairs of N Numbers. Let R be the relation in $N \times N$ which is defined by (a,b) is related to (c,d) which we shall write as $(a,b) \cdot (c,d)$ if and only if $ad = bc$. prove that R is an equivalence relation

Solution

$(a,b) \simeq (c,d)$ since $ab = ba$ hence R is reflexive.
Suppose $(a,b) \simeq (c,d)$ then $ad = bc$ which implies $cb = da$
Thus $(c,d) \simeq (a,b)$ and R is symmetric. Now
Suppose $(a,b) \simeq (c,d)$ and $(c,d) \simeq (e,f)$

Then $ad = bc$ and $cf = de$ Thus $(ad)(cf) = (bc)(de)$ and by cancelling from both sides $af = be$.

Thus $(a,b) \simeq (e,f)$ and hence R is Transitive

Accordingly R is an equivalence relation

04/03/22

Matrices & Relations

Adjacency Matrix :-

Let A & B be finite sets of order m and n respectively and let r be relation from A to B then, r can be represented by a matrix R called the Adjacency matrix

Let $A = \{a_1, a_2 \dots a_n\}$ and $B = \{b_1, b_2 \dots b_n\}$

then R is an m×n matrix

$$R_{ij} = \begin{cases} 1 & \text{if } a_i r b_j \\ 0 & \text{otherwise} \end{cases}$$

Example

1) Let $A = \{2, 5, 6\}$ let r be relation $\{(2,2)\ (2,5)\ (5,6)\ (6,6)\}$ since r is a relation from A to A.

Sol :-

$$M_R = \begin{array}{c|c} & \begin{array}{ccc} 2 & 5 & 6 \end{array} \\ \begin{array}{c} 2 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

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2) Let $A = \{1, 2, 3, 4, 5\}$ and define r on A if $y = x+1$, we define $r^2 = rr$, $r^3 = r^2 r$ find a) r b) r^2 c) r^3 write the matrix relation for r and r^2

Solution :-

$r \Rightarrow y = x+1$

$r = \{(1,2)(2,3)(3,4)(4,5)\}$

$r^2 : r \cdot r$

$: \{(1,2)(2,3)(3,4)(4,5)\} \cdot \{(1,2)(2,3)(3,4)(4,5)\}$

$: \{(1,3)(2,4)(3,5)\}$

$r^3 : r^2 \cdot r$

$: \{(1,3)(2,4)(3,5)\} \cdot \{(1,2)(2,3)(3,4)(4,5)\}$

$: \{(1,4)(2,5)\}$

$$M_R : r : \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 0 & 1 & 0 \\ 4 & 0 & 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 0 & 0 & 0 \end{array}$$

Q) consider $A = \{1,2,3,4,5\}$ the relation

$r = \{(1,4)(2,1)(2,2)(2,3)(3,2)(4,3)(4,5)(5,1)\}$

Write the matrix relation for r and r^2

$r^2 = \{(1,4)(2,1)(2,2)(2,3)(3,2)(4,3)(4,5)(5,1)\} \cdot$

$\{(1,4)(2,1)(2,2)(2,3)(3,2)(4,3)(4,5)(5,1)\}$

$(2,4)(2,1)$

$: \{\cancel{(1,5)}\cancel{(2,4)}\ (1,3)(1,5)(2,2)(2,3)\cancel{(2,2)}$

$(3,1)(4,1)\}$

$(3,1)(3,2)(3,3)$

$: \{(1,3)(1,5)(2,4)(2,1)(2,2)(2,3)(4,2)\cancel{(4,1)}$ $(4,1)$

$(5,4)\}$

$$M_R\ r : \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 0 & 1 & 0 & 1 \\ 2 & 1 & 1 & 1 & 1 & 0 \\ 3 & 1 & 1 & 1 & 0 & 0 \\ 4 & 1 & 1 & 0 & 0 & 0 \end{array}$$

$$M_R \; r : \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$M_R : r^2 : \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Function

A function from a set A into set B is a relation from A into B such that each element of A is related to exactly one element of the set B. The set A is called the domain of the function and the set B is called the codomain. The phrase "is related to exactly one element of the set B" means that if $(a,b) \in f$ and $(a,c) \in f$, then $b = c$. We write $f : A \to B$. Further if $a \in A$ then the element in B which is assigned to a is called the image of a and is denoted by $f(a)$ which reads "f of a".

Example 1

Let f assign to each real number its square, that is for every real number x let $f(x) = x^2$. The domain & codomain of f are both real numbers, so we can write $f : R^{\#} \to R^{\#}$

eg: 2

If f is $f : A \to A$ The image of -3 is $9 \Rightarrow f(-3) = 9$

We call f as transformation on A.

Definition

Range of a function :-

Let f be a mapping of A into B, that is let $f : A \to B$. Each element in B need not appear as the image of an element in A. We define the range of to consists precisely of those elements in B which appear as the image of atleast one element in A. We denote the range of $f : A \to B$ by $f(A)$. Notice that $f(A)$ is subset of B

Example:

Let the function $f : R^{\#} \to R^{\#}$ be defined by the formala $f(x) = x^2$. Then the range of f consists of the positive real numbers and zero.

Definition - one - one - Function

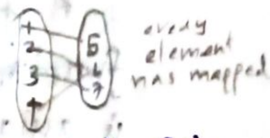
also called injective function.

Let f map A into B. Then f is called one - one function if different element in B are assigned to different elements in A (ie) $f(a) = f(a') \Rightarrow a = a'$ or equivalently $a \neq a' \Rightarrow f(a) \neq f(a')$

Example:-

Let $f : R^{\#} \to R^{\#}$ be defined by the formula $f(x) = x^2$. Then $f(-2) = 4 = f(2)$, 4 but $2 \neq -2 \Rightarrow f$ is not injective

Definition onto function



also known surjective function

Let f be a function of A into B, if $f(A) = B$ (ie) if every number of B appears as the image of atleast one element of A, then we say f is a function of A onto B.

Example 1:

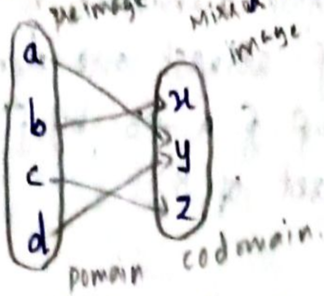
Let the function $f : R^{\#} \to R^{\#}$ be defined by the formula $f(x) = x^2$. Then f is not an onto function since the negative real numbers do not appear in the range of f (ie) no negative real number is the square of a real number.

Example 2:-

Let $A = \{a, b, c, d\}$

$B = \{x, y, z\}$

Let $f: A \to B$ defined by the diagram



Notice that $f(A) = \{x, y, z\} = B$ that is range of f is equal to the codomain B. Thus f maps A onto B (ie) f is an onto mapping.

Identify function [Definition]

Let A be any set. Let the function $f: A \to A$ be defined by the formula $f(x) = x$. (ie) Let f assign to each element in A the element itself. Then f is called the identify function. We denote this by 1 or 1_A.

Definition: A Map which is both surjective and injective is said to be bijective map.

Definition

A function $f: A \to A$ be defined be a constant function of the same element $b \in B$ ~~let f~~ is assigned to each element in A. in other words f is a constant function if the range of f is a singleton.

Example:-

Let $f: R \to R$ defined by $f(x) = 4$ for all $x \in R$

Then f is a constant function

Product or composite of functions

Definition

Let $f: A \to B$ $g: B \to C$ be any 2 functions Then the composite of f & g, denoted by gof, is a function from A into C defined by $(g \circ f)(a) = g(f(a))$ for all $a \in A$.

07/03/21

Inverse function

Definition:-

Let $f : A \to A$ if there exists a function $g : A \to A$ such that $g \circ f = f \circ g = i_A$ (where i_A denotes identify function} then g is called the inverse of the function f denoted by f^{-1}

Example:-

Let $g : R \to R$ be defined by $g(x) = x^2$

Find $g^{-1}(4), g^{-1}(0), g^{-1}(-1)$

Solution:

$g^{-1}(4) = \{2, -2\}$

$g^{-1}(0) = 0$

$g^{-1}(-1) = \phi$

Inverse function (another definition)

Let $f : A \to B$ be a bijection. Then the map $f^{-1} : B \to A$ which associates with each element $b \in B$ a unique element $a \in A$ such that $f(a) = b$ is called an inverse map of f.

Theorems on function

Let $f : A \to B$ and $g : B \to C$ be maps. then

i) f is 1-1, g is 1-1 $\Rightarrow g \circ f$ is 1-1

ii) f is onto, g is ont $\Rightarrow g \circ f$ is onto

iii) $g \circ f$ is onto $\to g$ is onto

iv) $g \circ f$ is onto $\Rightarrow g$ is onto.

Proof:-

i) To prove $g \circ f : A \to C$ is 1-1

We have to show that $(g \circ f)(a_1) = (g \circ f)(a_2)$

$\Rightarrow a_1 = a_2$. $a_1, a_2 \in A$

Now $(g \circ f)(a_1) = (g \circ f)(a_2)$

$g(f(a_1)) = g(f(a_2))$

$\Rightarrow g(b_1) = g(b_2)$ where

$b_1 = f(a_1)$ and $b_2 = f(a_2)$

$\Rightarrow b_1 = b_2$ [$\because g$ is 1-1]

$f(a_1) = f(a_2)$

$\Rightarrow a_1 = a_2$ [$\because f$ is 1-1)

ii) To prove $g \circ f : A \rightarrow C$ is onto

Let $c \in C$ be arbitary, since g is onto there exist an element $b \in B$ that $g(b) = c$, since f is onto, there exists an element $a \in A$ such $f(a) = b$

$\therefore (g \circ f)(a) = g(f(a)) = g(b) = c$

Thus for any $c \in C$ there exists atleast one $a \in A$ such that $(g \circ f)(a) = c$

$\therefore g \circ f$ is onto.

iii) To prove $f : A \rightarrow B$ is 1-1.

We have to show that $f(a_1) = f(a_2) \Rightarrow a_1 = a_2$
$a_1, a_2 \in A$

Now $f(a_1) = f(a_2) \Rightarrow g(f(a_1)) = g f(a_2))$

$g \circ f)(a_1) = (g \circ f)(a_2)$

$\Rightarrow a_1 = a_2$ [$\because g \circ f$ is 1-1]

iv) To prove $g : B \rightarrow C$ is onto.

Let $c \in C$, be arbitrary, since $g \circ f : A \rightarrow C$ onto, there exists $a \in A$ such that $(g \circ f)(a) = c$ (ie) $g f(a) = c$ (or) $g(b) = c$, where $b = f(a) \in B$.

Hence for $c \in C$ there exists $b \in B$ such that $g(b) = c$.

$\therefore g$ is onto

Note: from (i)(ii) we observe that if $f: A \to B$, $g: B \to C$ are bijective $g \circ f: A \to C$ is also bijective

Theorem 2: Let $f: A \to B$ be a bijection

Then $f^{-1}: B \to A$ is also a bijection

proof: Since f is bijective, $f^{-1}: B \to A$ is defined

claim: f^{-1} is 1-1

(ie) $f^{-1}(b_1) = f^{-1}(b_2) \Rightarrow b_1 = b_2$, $b_1, b_2 \in B$.

Let $f^{-1}(b_1) = a_1$, $f^{-1}(b_2) = a_2$

Then $b_1 = f(a_1)$ and $b_2 = f(b_2)$

Now $f^{-1}(b_1) = f^{-1}(b_2) \Rightarrow a_1 = a_2$

$\Rightarrow f(a_1) = f(a_2)$ $\quad \therefore$ (f is a map)

$\Rightarrow b_1 = b_2$

Hence f^{-1} is 1-1

To show $f^{-1}: B \to A$ is onto

Let $a \in A$ be arbitrary. Since f is a map, there exists $b \in B$ such that $f(a) = b$ (ie) $a = f^{-1}(b)$.

$\therefore a \in A$, there exists $b \in b$ such that which $f^{-1}(b) = a$ (ie) f^{-1} is onto.

Hence f^{-1} is bijective

Theorem 3: Let $f: A \to B$ be a bijection

Then $f^{-1} \circ f = I_A$ and $f \circ f^{-1} = I_B$.

proof: Let $f: A \to B$ be a bijection

Then $f^{-1} \circ f$ and I_A are maps from A to A

For each $a \in A$

$(f^{-1} \circ f)(a) = f^{-1}(f(a)) = f^{-1}(b)$

where $b = f(a)$ (ie) $f^{-1}(b) = a$

$\therefore (f^{-1} \circ f)(a) = a \; \forall a \in A$

Hence $f^{-1} \circ f = I_A$ similarly $f \circ f^{-1} = I_B$.

Theorem 4 :

Let $f: A \to B$ and $g: B \to C$ be bijective

Then $(gof)^{-1} = f^{-1} o g^{-1}$ (reversal law for inverse function)

Proof : $f: A \to B$ and $g: B \to C$ are bijective

Hence $gof : A \to C$ is bijective and $(gof)^{-1} : C \to A$ exists.

Also $f^{-1} : B \to A$ and $g^{-1} : C \to B$ exist

$\therefore f^{-1} o g^{-1} : C \to A$

Thus $(gof)^{-1}$ and $f^{-1} o g^{-1}$ are both functions from C to A

Also we know

i) $fog^{-1} = 1_B \cdot f^{-1} of = 1_A$

ii) $gog^{-1} = 1_A \cdot g^{-1} og = 1_B$ and

iii) product of any function with the identify function is function itself

Hence $(f^{-1} o g^{-1}) o (gof) = f^{-1} o [(g^{-1} o g) o f]$ [o is Associati

$= f^{-1} o (1_B o f)$

$= f^{-1} o f = 1_A$.

$(gof) o (f^{-1} o g^{-1}) = go [(f o f^{-1}) o g^{-1}]$

$= go [1_B o g^{-1})$

$= gog^{-1} = 1_A$

By theorem 3, it follows that

$(gof)^{-1} = f^{-1} o g^{-1}$

Theorem 5

Let $f: A \to B$ be a map Then the set of all non-empty subsets of A which are inverse images of elements of B form a partition of A.

Proof : Let $a \in A$ be arbitrary and let $f(a) = b$,
Then $a \in f^{-1}(b)$.

Hence any element of A is an element $f^{-1}(b)$ for some $b \in B$.

Again $a \in A \Rightarrow a \in f^{-1}(b)$ for some $b \in B$.

$\Rightarrow a \in \bigcup_{b \in B} f^{-1}(b)$ for some $b \in B$

Hence $A \subseteq \bigcup_{b \in B} f^{-1}(b)$

Also $f^{-1}(b) \in A \Rightarrow \bigcup_{b \in B} f^{-1}(b) \subseteq A$

$\therefore A = \bigcup_{b \in B} f^{-1}(b)$

To show that $f^{-1}(b_1) \cup f^{-1}(b_2) = \phi$ if $b_1 \neq b_2$.

suppose $f^{-1}(b_1) \cup f^{-1}(b_2) = \phi$ for $b_1 \neq b_2$

Then there exist an element $x \in f^{-1}(b_1) \cup f^{-1}(b_2)$.

$\therefore f(x) = b_1$ and $f(x) = b_2$. since f is a map this is possible only $b_1 = b_2$ which is contradiction

$\therefore f^{-1}(b_1) \cup f^{-1}(b_2) = \phi$ if $b_1 = b_2$

Hence the theorem.

8/3/22

1) Let $f: R \to R$ & $g: R \to R$ defined by $f(x) = 4x - 1$
$g(x) = \cos x$ find fog & gof

$fog = f(g(x))$

$= f(\cos x)$

$= 4\cos x - 1$

$gof = g(f(x))$

$= g(4x - 1)$

$= \cos 4x - 1$ //.

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2) $f(x) = 2x - 1$ $g(x) = x^2 - 2$

$fog = f(g(x))$

$= f(x^2 - x)$

$= 2(x^2 - \cancel{2}) - 1 \quad = 2x^2 - 4 - 1$

$= 2x^2 - 5$ //

$gof = g(f(x))$

$= g(2x-1)$

$= (2x-1)^2 - 2$

$= 4x^2 - 4x + 1 - 2$

$= 4x^2 - 4x - 1$ //

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3) Let $A = \{1, 2, 3\}$ & f, g, h are functional from x to x by $f = \{(1,2)(2,3)(3,1)\}$

$g = \{(1,2)(2,1)(3,3)\}$ $h = \{(1,1)(2,2)(3,1)\}$

and $s = \{(1,1)(2,2)(3,3)\}$ find fog, gof, soh, hog

$fog = \{(1,2)(2,3)(3,1)\} \cdot \{(1,2)(2,1)(3,3)\}$

$= \{(1,1)(2,3)(3,2)\}$

$gof = \{(1,2)(2,1)(3,3)\} \cdot \{(1,2)(2,3)(3,1)\}$

$= \{(1,3)(2,2)(3,1)\}$

$soh = \{(1,1)(2,2)(3,3)\} \cdot \{(1,1)(2,2)(3,1)\}$

$= \{(1,1)(2,2)(3,1)\}$

$hog = \{(1,1)(2,2)(3,1)\} \cdot \{(1,2)(2,1)(3,3)\}$

$= \{(1,2)(2,1)(3,2)\}$